מבנה מחשבים - תרגיל #2

אריאל סטולרמן
קבוצה 02

(1)

1. a + a'b + a'b'c + a'b'c'd + ... = a + a'(b + b'(c + c'(d + ... =
   a + b + b'(c + c'(d + ... = ... = **a + b + c + d + ...**
2. (x' + xyz') + (x' + xyz')(x + x'y'z) =
   x' + yz' + (x' + yz')(x + y'z) = (x' + yz')(1 + x + y'z) =
   x' + yz'
3. (x + y)(y + z) + xz' = xy + xz + yy + yz + xz' =
   xy + y + x(z + z') + yz = y + x + yz = **x + y**

(2)

נמצא את המשלים של הביטויים הבאים, ונצמצם את התוצאה (לא את המקור):

1. x'(y' + z')(x + y + z'):
   [x'(y' + z')(x + y + z')]' = x'' + (y' + z')' + (x + y + z')' =
   x + y''z'' + x'y'z'' = x + yz + x'y'z = x + z(y + y'x') =
   x + z(y + x') = x + x'z + zy = x + z + zy = **x + z**
2. (x + yz')(y + x'z')(z + x'y'):
   [(x + yz')(y + x'z')(z + x'y')]' = (x + yz')' + (y + x'z')' + (z + x'y')' =
   x'(y' + z'') + y'(x'' + z'') + z'(x'' + y'') =
   x'(y' + z) + y'(x + z) + z'(x + y) = x'y' + x'z + xy' + y'z + xz' + yz' =
   y'(x' + x + z) + x'z + xz' + yz' = y'•1 + yz' + x'z + xz' =
   (y' + z')(1 + y) + x'z + xz' = y' + z' + x'z + xz' =
   y' + z'•1 + zx' + xz' = y' + (z' + x')(z + 1) + xz' =
   y' + z' + x' + xz' = y' + z' + x'•1 + xz' =
   y' + z' + (x' + z')(1 + x) = y' + z' + x' + z' =
   x' + y' + z'
3. (a + b'c')(b + a'c')(c + a'b'):
   [(a + b'c')(b + a'c')(c + a'b')]' = (a + b'c')' + (b + a'c')' + (c + a'b')' =
   a'(b'' + c'') + b'(a'' + c'') + c'(a'' + b'') =
   a'b + a'c + ab' + b'c + ac' + bc' = <u>ab' + a'c + b'c</u> + <u>ac' + a'b + bc'</u> =
   ab' + a'c + ac' + a'b

(3)

1. ab + a'b' + bc = ab + a'b' + a'c:
   ab + a'b' + bc = ab + <u>b'a' + bc</u> = ab + <u>b'a' + bc + a'c</u> = <u>ab + a'c + bc</u> + a'b' =
   <u>ab + a'c</u> + a'b' = ab + a'b' + a'c ■ {xy + x'z + yz = xy + x'z}

2. $xy + x'z + yz = xy + x'z$: מיידי מהזהות לעיל, נוכיח אחרת

x	y	z	x'	xy	x'z	yz	xy + x'z + yz	xy + x'z
0	0	0	1	0	0	0	**0**	**0**
0	0	1	1	0	1	0	**1**	**1**
0	1	0	1	0	0	0	**0**	**0**
0	1	1	1	0	1	1	**1**	**1**
1	0	0	0	0	0	0	**0**	**0**
1	0	1	0	0	0	0	**0**	**0**
1	1	0	0	1	0	0	**1**	**1**
1	1	1	0	1	0	1	**1**	**1**

➔ $xy + x'z + yz = xy + x'z$ ■

3. $xy + x'z + F(x,y,z)yz = xy + x'z$

let $F(x,y,z) = 0$: $xy + x'z + F(x,y,z)yz = xy + x'z$ – מיידי

let $F(x,y,z) = 1$: $xy + x'z + F(x,y,z)yz = xy + x'z + yz$ – הוכח לעיל

➔ $\forall F: xy + x'z + F(x,y,z)yz = xy + x'z$ ■

דרך נוספת:

$xy + x'z + F(x,y,z)yz = xy + x'z + yz + F(x,y,z)yz = xy + x'z + yz(1 + F(x,y,z))$

$= xy + x'z + yz = xy + x'z + yz$ ■

(4)

1. $F(x,y,z):=(xy)\ XOR\ (yz)\ XOR\ (1)$

נראה כי $\{F,\ 0,\ 1\}$ היא מערכת אוניברסלית:

נממש את $\{NOT,\ AND\}$:

$\underline{NOT(x):=\ F(x,1,0)}$:

$F(x,1,0) = (x)\ XOR\ (0)\ XOR\ (1) = (x)\ XOR\ (1) = x'$

$\underline{AND(x,y):=\ F(x,\ y,\ 0)}$:

$NOT(F(x,\ y,\ 1)) = NOT((xy)\ XOR\ (y \bullet 0)\ XOR\ (1)) = NOT((xy)\ XOR\ (1))$

x	y	(xy) XOR (1)	NOT((xy) XOR (1))	AND(x,y)
0	0	1	**0**	**0**
0	1	1	**0**	**0**
1	0	1	**0**	**0**
1	1	0	**1**	**1**

➔ $\{F,\ 0,\ 1\}$ היא מערכת אוניברסלית ■

$\underline{a\ XOR\ b:=\ F(a,\ 1,\ F(b,\ 1,\ 0)}$:

$a\ XOR\ b = F(a,\ 1,\ F(b,\ 1,\ 0)) = F(a,\ 1,\ b') = (a \bullet 1)\ XOR\ (1 \bullet b')\ XOR\ (1) =$

$a\ XOR\ [b'\ XOR\ (1)] = a\ XOR\ (b' \bullet 0 + b'' \bullet 1) = a\ XOR\ b$ ■

2. $F(w,x,y,z) = \Sigma(4, 5, 13)$ ➔

#	x	y	z	w	F(x,y,z,w)
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

$F(x, y, z, w) = x'yz'w' + x'yz'w + xyz'w =$
$x'yz'w' + yz'w(x' + x) = x'yz'w' + yz'w =$
$yz'(x'w' + w) = yz'(w + x') =$ **x'yz' + yz'w**

נראה כי $\{F, 1\}$ היא מערכת אוניברסלית:

נממש את $\{NOT, AND\}$:

$NOT(x) := F(x, 1, x, 1):$

$F(x, 1, x, 1) = x'\bullet1\bullet x' + 1\bullet x'\bullet1 = x'x' + x' =$ **x'**

$AND(x,y) := F(F(x, 1, x, 1), y, F(x, 1, x, 1), 1):$

$F(F(x, 1, x, 1), y, F(x, 1, x, 1), 1) =$

$F(x', y, x', 1) = x''yx'' + yx''\bullet1 =$

$xyx + yx = xy + xy =$ **xy**

$\{F, 1\}$ היא מערכת אוניברסלית ■ ➔

3.

נתונות הפונקציות:

$f1(x,y,z) = (x'y) \text{ XOR } (xy) \text{ XOR } (yz) \text{ XOR } (y'z')$

$f2(x,y,z) = (x) \text{ XOR } (y) \text{ XOR } (z)$

ניצור בעזרתן את הפונקציה:

$g(x,y,z) = xyz' + xy'z + x'yz + x'y'z'$

$f1(x,y,z) = [(x'y) \text{ XOR } (xy)] \text{ XOR } [(yz) \text{ XOR } (y'z')] = [(x'y)(xy)' + (x'y)'(xy)] \text{ XOR } [(yz)(y'z')' + (yz)'(y'z')] = [x'y(x' + y') + xy(x + y')] \text{ XOR } [yz(y + z) + y'z'(y' + z')] = [x'yx' + x'yy' + xyx + xyy'] \text{ XOR } [yzy + yzz + y'z'y' + y'z'z'] = [x'y + xy] \text{ XOR } [yz + y'z'] = y \text{ XOR } (yz + y'z') = y(yz + y'z')' + y'(yz + y'z') = y(yz)'(y'z')' + y'yz + y'y'z' = y(y' + z')(y + z) + y'z' = yz'(y + z) + y'z' = yz'y + yz'z + y'z' = yz' + y'z' = z'(y + y') =$ **z'**

$f2(x,y,z) = [(x) \text{ XOR } (y)] \text{ XOR } (z) = [x'y + xy'] \text{ XOR } (z) = z'(x'y + xy') + z(x'y + xy')' = x'yz' + xy'z' + z(x'y)'(xy')' = x'yz' + xy'z' + z(x + y')(x' + y) = x'yz' + xy'z' + z(xx' + xy + x'y' + yy') =$ **x'yz' + xy'z' + xyz + x'y'z**

$g(x,y,z) := f2(x,y,f1(x,y,z)):$

$f2(x,y,f1(x,y,z)) = f2(x,y,z') = x'yz'' + xy'z'' + xyz' + x'y'z' =$

$xyz' + xy'z + x'yz + x'y'z'$ ■

(5)

נתונה הפונקציה:

F(W,X,Y,Z) = X'W(Y'Z + X') + [X(XOR)Y]

נמצא לה הצגה קנונית לפי SOP ו-POS.

תחילה נפשט את הפונקציה:

F(w,x,y,z) = x'w(y'z + x') + (x) XOR (y) = x'wy'z + x'wx' + xy' + x'y =

x'wy'z + x'w + xy' + x'y

#	w	x	y	z	x'	y'	x'wy'z	x'w	xy'	x'y	F(w,x,y,z)
0	0	0	0	0	1	1	0	0	0	0	0
1	0	0	0	1	1	1	0	0	0	0	0
2	0	0	1	0	1	0	0	0	0	1	1
3	0	0	1	1	1	0	0	0	0	1	1
4	0	1	0	0	0	1	0	0	1	0	1
5	0	1	0	1	0	1	0	0	1	0	1
6	0	1	1	0	0	0	0	0	0	0	0
7	0	1	1	1	0	0	0	0	0	0	0
8	1	0	0	0	1	1	0	1	0	0	1
9	1	0	0	1	1	1	1	1	0	0	1
10	1	0	1	0	1	0	0	1	0	1	1
11	1	0	1	1	1	0	0	1	0	1	1
12	1	1	0	0	0	1	0	0	1	0	1
13	1	1	0	1	0	1	0	0	1	0	1
14	1	1	1	0	0	0	0	0	0	0	0
15	1	1	1	1	0	0	0	0	0	0	0

SOP: F(w,x,y,z) = Σ(2,3,4,5,8,9,10,11,12,13) =

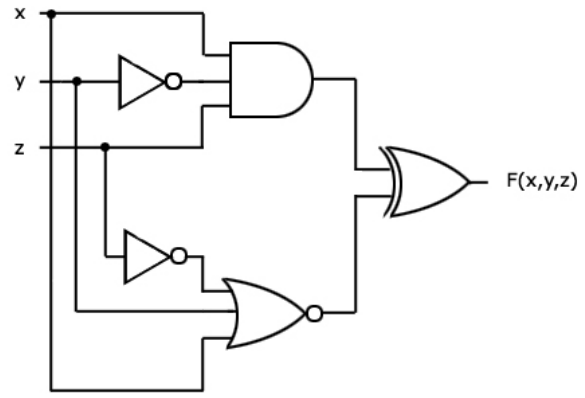
w'x'yz'+w'x'yz+w'xy'z'+w'xy'z+wx'y'z'+wx'y'z+wx'yz'+wx'yz+wxy'z'+wxy'z

POS: F(w,x,y,z) = Π(0,1,6,7,14,15) =

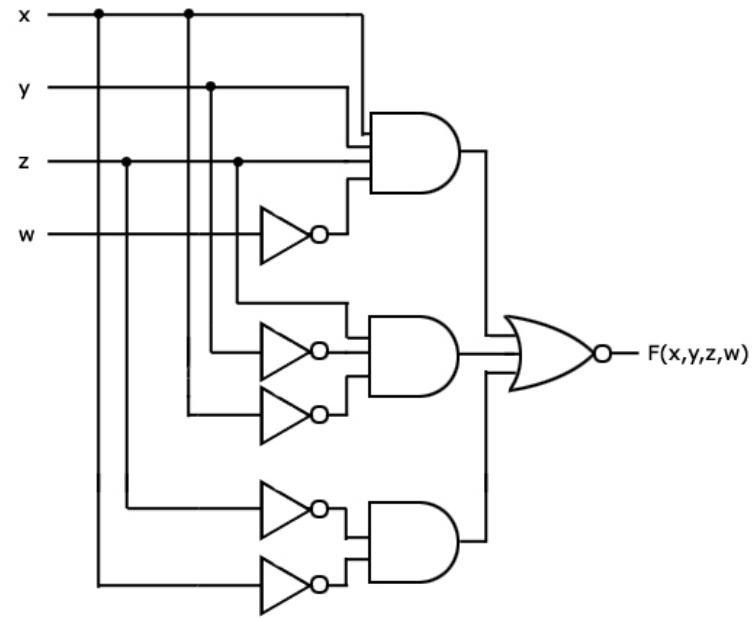
(w+x+y+z)(w+x+y+z')(w+x'+y'+z)(w+x'+y'+z')(w'+x'+y'+z)(w'+x'+y'+z')

(6)

F(x,y,z) = (xy'z) XOR (x+y+z')':



F(x,y,z,w) = (x'y'z + xyzw' + x'z')':



F(x,y,z,w) = (x + y + z')(x' + y' + w)z + (xy + z)':