1. Show that P is closed under polynomial-time Cook reductions.

2. A k-CNF formula is *NAE-satisfiable* if it can be satisfied in such a way that each clause has at least one true literal and at least one false literal (NAE stands for Not-All-Equal). For example, the clause $(x_1 \vee x_2 \vee \overline{x_3})$ is NAE-satisfied by $x_1 = T, x_2 = T, x_3 = T$, and not by $x_1 = T, x_2 = T, x_3 = F$. Let NAE-$k$SAT be the language of all k-CNF NAE-satisfiable formulas.

   (a) Show that NAE-3SAT is NP-complete.
   Suggestion: Show that 3SAT $\leq_p$ NAE-4SAT $\leq_p$ NAE-3SAT.

   (b) Is EVEN-NAE-3SAT= $\{ \phi \mid \phi$ is a 3-CNF formula with an even number of NAE-satisfying assignments$\}$ NP-hard?

3. Recall that a graph is k-colorable if its vertices can be colored using up to k different colors in such a way that any two adjacent vertices have different colors. For any $k \in \mathbb{N}$ define the language k-Col $= \{ G \mid G$ is k-colorable $\}$.

   (a) Show that a graph is 2-colorable if and only if it has no cycle of odd length, and deduce that 2-Col is in P.

   (b) Prove that 3-Col is NP-complete.
   Hint: Reduce from NAE-3SAT. Given a formula generate a graph as follows: associate a vertex to each *literal*. Connect all these vertices to a vertex w and connect each variable vertex to its negation. Then, add a triangle for each clause and connect its vertices to the corresponding literals.

   (c) Deduce that the following languages are NP-complete.

      i. 2009-Col.

      ii. Coloring $= \{ (G, k) \mid G$ is k-colorable $\}$.

      iii. CliqueCover $= \{ (G, k) \mid$ the vertices of G can be partitioned into k sets, so that each set induces a clique$\}$.

4. A polynomial-time reduction f from a language $L \in$ NP to a language $L' \in$ NP is *parsimonious* if the number of witnesses of x is equal to the number of witnesses of $f(x)$. Show a polynomial-time parsimonious reduction from SAT to 3SAT (where by a witness for SAT/3SAT we mean a satisfying assignment).