Computational Models - Exercise 3
8/12/08

1.
   a. Write a Turing machine that computes the function $f(w) = 101w$. Give a complete formal description of the machine. ($\Sigma=\{0,1\}$)
   b. Write a Counter machine that computes the following function: $f(n) = \lfloor n/2 \rfloor$. You may use any number of registers. Give a complete formal description of the machine (i.e., the number of registers and a finite sequence of instructions as explained in recitation and class, see definition 1 in chapter 3.2 in L1.pdf in the site of the course).

2.
   a. Write a Turing machine that decides the language $L_2 = \{1^k \mid k=3^n, n\in N\}$ ($\Sigma=\{1\}$) . Don't give a formal description, only describe the way the machine works (in the spirit of what was done in recitation).
   b. Write a counter machine that computes the function $f(n) = n^2$. Don't give a formal description, only describe the way the machine works (in the spirit of what was done in recitation).

3. We define the following variations of Turing machines:
   a. Machines of the type TM' that are similar to regular TMs, except that $|Q|<2008$ and $|\Gamma|<2008$. In addition the head of the tape can move up to 2008 cells to the right or left at one step. For example: $\delta(q_5,a)=q_{17},b,R28$ means that if the tape is at state q_5 and reads the character a, it shall move to state q_{17} write b on the tape and move 28 cells to the right. Let L(TM') be the languages accepted by TM' machines. Decide and prove whether $L(TM')\subset RE$, $RE\subset L(TM')$ or $L(TM')=RE$.
   b. Machines of the type TM'' that are similar to regular TMs, except that they can move to the right or left any number of cells. Decide and prove whether $RE=L(TM'')$ or $RE\subset L(TM'')$.

4. For the following languages determine whether they belong to *R*, *RE/R*, *co-RE/R*, or none of the above, and prove correctness:
   a. Input: A Turing machine *M* and a natural number *K*.
      Question: Is there an input for which *M* halts in less than *K* steps.
   b. Input: A Turing machine *M* and a string *x*.
      Question: Does M write '1' on the tape during execution on *x*.
   c. Input: A Turing machine *M*
      Question: is *M* a Turing machine such that for any input *x* it does not reach the $|x|+1000$ cell. Hint: Use configurations and computational histories.

5. The accepting language $Accept = \{\langle p,x\rangle \mid p \text{ is a program that accepts } x\}$ is in RE\R. Prove that $L\in RE$ iff $L\leq_m Accept$.

6. Prove or disprove:
   a. The class *RE* is closed under union and intersection
   b. The class *co-RE* is closed under union and intersection

c. If the language A is not decidable and $A \leq_m A^c$ (The complement of A), then $A \notin RE$ and $A \notin co\text{-}RE$

7. Let $L_1, L_2 \subseteq \{0,1\}^*$ be RE languages such that $L_1 \cup L_2 = \{0,1\}^*$ and $L_1 \cap L_2 \neq \phi$. Show that $L_1 \leq_m L_1 \cap L_2$.