Numerical Analysis 1

Assignment 7

1. Given

x	0	π
$f(x) = \sin(x)$	0	0
$f'(x) = \cos(x)$	1	-1
$f''(x) = -\sin(x)$	0	0

   (a) Write down Hermite interpolation polynomial.

   (b) Give an error bound in $[0, \pi]$ and $[-\frac{\pi}{2}, \frac{3\pi}{2}]$.

   (c) Draw your interpolation versus $sin(x)$.

2. For the matrices
$$A = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 3 & 0 \\ 2 & 0 & 2 \\ 0 & 6 & 0 \end{bmatrix}$$

   calculate the 1, 2 and ∞ norms.

3. Prove that
$$\|A\|_1 = \max_{1 \leq j \leq n} \sum_{i=1}^{n} |a_{ij}| = \|A^T\|_\infty$$

4. For an induced norm, $\|\cdot\|$, prove that $\rho(A) \leq \|A\|$ and that $\rho(A) \leq \|A^2\|^{1/2}$, where ρ is the spectral radius
$$\rho(A) = \max \{|\lambda_i|\}_{\lambda_i \text{ eigenvalues of } A} \cdot$$

5. Give an example of a 3×3 system for which the Jacobi iterations converge even though the infinity norm of B is greater than 1.

6. Consider the following system

$$\begin{array}{ccccccc} 6x & + & 2y & + & 2z & = & -2 \\ 6x & + & 2y/3 & + & z/3 & = & 0.1 \\ x & + & 2y & + & -z & = & 0 \end{array}$$

   by Jacobi and by Gauss-Seidel methods with 10^{-2} accuracy, i.e. $\|x^{(k)} - \bar{x}\|_\infty < 10^{-2}$. Use zero vector as initial guess.

7. Consider the following system of equations:

$$\begin{array}{rcrcrcl}
6x & + & 2y & + & 2z & = & 10 \\
2x & + & 5y & + & z & = & 12 \\
x & + & 2y & - & 4z & = & 5
\end{array}$$

with the exact solution $x = (1, 2, 0)^T$.

(a) Write down the Gauss-Jacobi method corresponding (specifically) to this system.

(b) Repeat part (a) for the Gauss-Seidel method.

(c) Prove that the Gauss-Jacobi and Gauss-Seidel iterations converge for this system.

(d) Use Matlab to calculate the spectral radii (i.e., the ρ's) corresponding to the Gauss-Jacobi and Gauss-Seidel methods for this system.

(e) Find out how many iterations are needed using both methods in order to achieve $\|x^{(k)} - \bar{x}\|_\infty < 10^{-3}$? For the initial guess set $x^{(0)} = 0$.